SEPARATION OF VARIABLES AT ANY RANK: FROM SPIN CHAINS TO FISHNET CFT

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based on

20xx.xxxx [Gromov, FLM, Ryan, Volin]
2003.05811 [FLM, Preti]
1910.13442 [Gromov, FLM, Ryan, Volin]
1907.03788 [Cavaglia, Gromov, FLM]
+ in progress [Cavaglia, Gromov, FLM, …]

Motivation: develop new methods to compute correlators in N=4 SYM

Should exist a basis where wavefunctions factorize

$$\langle x|\Psi\rangle \sim Q(x_1)Q(x_2)\ldots Q(x_N)$$

Separation of Variables (SoV)

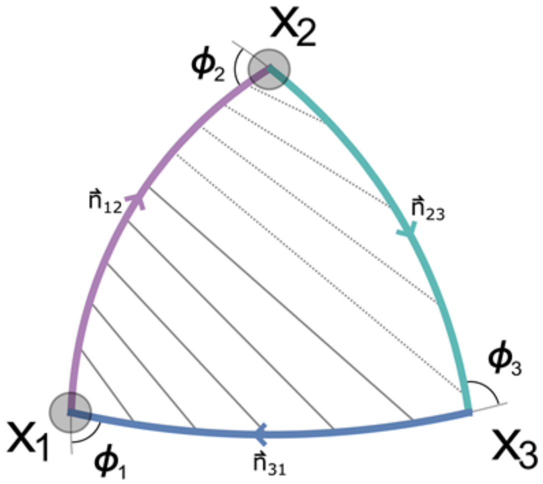
We know exact Q's from Quantum Spectral Curve equations for spectrum [Gromov, Kazakov, Leurent, Volin 13]

Goal: write correlators in terms of Q's

First all-loop example:
3 cusps + scalars in ladders limit,
resum all wrapppings

$$C_{123}^{\bullet\bullet\circ} = \frac{\langle q_1\, q_2\, e^{-\phi_3 u}\rangle}{\sqrt{\langle q_1^2\rangle\langle q_2^2\rangle}}$$

[Cavaglia, Gromov, FLM 18]
extension: [McGovern 20]



Need to understand and develop SoV

For scalar products we need measure

In GL(2)-type models:

$$\langle \Psi_B | \Psi_A \rangle = \int d^L\mathbf{x} \left(\underbrace{\prod_{i=1}^{L} Q^{(A)}(x_i)}_{\text{state } A} \right) \underbrace{M(\mathbf{x})}_{\text{measure}} \left(\underbrace{\prod_{i=1}^{L} Q^{(B)}(x_i)}_{\text{state } B} \right)$$

e.g. for non-compact s=½ spin chain

$$M(\mathbf{x}) = \frac{\prod_{j<k} (e^{2\pi x_j} - e^{2\pi x_k})(x_j - x_k)}{\prod_{j,k} (1 + e^{2\pi(x_j - \theta_k)})}$$

[Sklyanin]
[Derkachov Korchemsky Manashov 02]

Higher rank GL(N) models are complicated.
Only recently understood how to factorise wave functions

[Sklyanin 92] [Smirnov 2000]
[Gromov FLM Sizov 16] [Maillet Niccoli 18] [Ryan Volin 18]
[Liashyk Slavnov 18] [Derkachov Valinevich 19]

Measure was not known at all, except in classical limit [Smirnov Zeitlin 02]

Focus of this talk – finding the measure

Plan

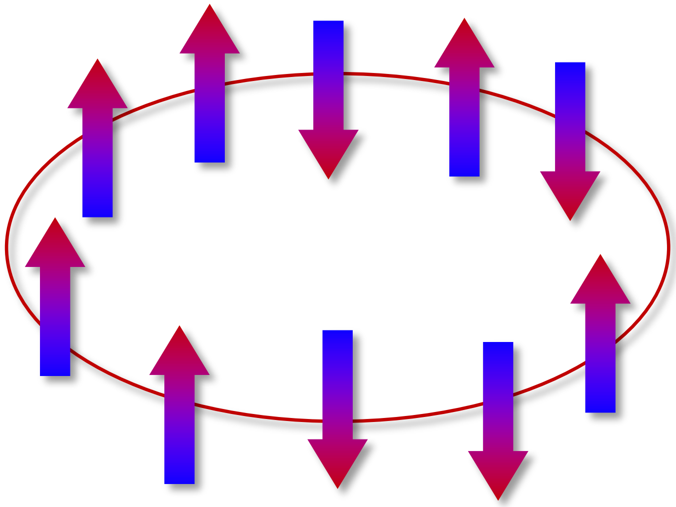
- Compact SU(N) spin chains [Gromov, FLM, Ryan, Volin 19]
- Noncompact SL(N) spin chains [Cavaglia, Gromov, FLM 19 Gromov, FLM, Ryan, Volin to appear]
- Fishnet theory & speculations

Chronologically first

COMPACT SPIN CHAINS

SU(N) spin chains

Full Hilbert space for L sites is $\underbrace{\mathbb{C}^N \otimes \mathbb{C}^N \otimes \cdots \otimes \mathbb{C}^N}_{L \text{ times}}$



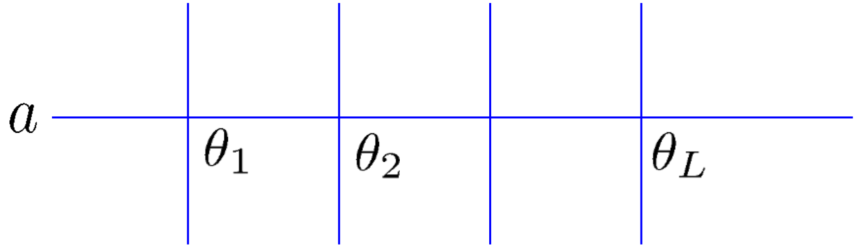
$$H = \sum_{n=1}^{L} (1 - P_{n,n+1})$$

(+ boundary terms, i.e. twist)

Monodromy matrix:

$$T(u) = R_{a1}(u - \theta_1) \dots R_{aL}(u - \theta_L) g$$

$$R_{12}(u) = (u - \tfrac{i}{2}) + i P_{12}$$



We take generic inhomogeneities θ_n and diagonal twist $g = \mathrm{diag}(\lambda_1, \dots, \lambda_{\mathrm{N}})$

Transfer matrix $\mathrm{Tr}_a\, T(u) = \sum_{n=0}^{L} T_n u^n$ gives commuting integrals of motion

Wavefunctions for spin chains

$$\langle x|\Psi\rangle = \prod_k Q_1(x_k) \qquad Q_1 = e^{\phi_1 u}\prod_{j=1}^{N_u}(u-u_j)$$

$\langle x|$ = eigenstates of operator $B(u) = \prod(u-x_k)$ $\qquad [B(u),B(v)]=0$

SU(2): $\quad T(u) = \begin{pmatrix} A(u) & B(u) \\ C(u) & D(u) \end{pmatrix} \qquad x_k = \theta_k \pm i/2, \quad k=1,\dots L$

SU(N): B is a polynomial in elements of T [Smirnov 2000] [Gromov, FLM, Sizov 16]

Can find spectrum of x, also build states nicely

$$|\Psi\rangle = B(u_1)B(u_2)\dots B(u_M)|0\rangle$$ [Gromov, FLM, Sizov 16]

$$T \to T^{\rm good} = KTK^{-1}$$
$$B \to B^{\rm good}$$

Proved in [Ryan, Volin 18] , connected with another way to build x [Maillet Niccoli 18-20]

SU(3): [Lyashik, Slavnov 18]

Overlaps look complicated, can we compute them indirectly?

SU(2) spin chain

Idea: orthogonality of states must imply same for Qs

$$Q_1 = e^{u\phi}\prod_{k=1}^{M}(u-u_k) \quad Q_\theta = \prod_{n=1}^{L}(u-\theta_n)$$

$$\tau_1 = 2\cos\phi\, u^L + \sum_{n=0}^{L-1} I_n u^n$$

$$f^\pm = f(u \pm i/2), \quad f^{[a]} = f(u+ia/2)$$

Baxter equation $Q_\theta^- Q_1^{++} + Q_\theta^+ Q_1^{--} - \tau_1 Q_1 = 0$

can be written as

$$\hat{O}\circ Q_1 = 0 \qquad \hat{O} = \frac{1}{Q_\theta^+}D^2 + \frac{1}{Q_\theta^-}D^{-2} - \frac{\tau_1}{Q_\theta^+ Q_\theta^-}$$

Key property: self-adjointness

$$\langle f\hat{O}g\rangle = \langle g\hat{O}f\rangle$$

$$\langle f\rangle = \oint du\, f(u)$$

$$\langle f\hat{O}g\rangle = \oint du\, f\left[\frac{g^{++}}{Q_\theta^+} + \frac{g^{--}}{Q_\theta^-} - \frac{\tau_1}{Q_\theta^+ Q_\theta^-}g\right]$$

$u \to u - i$

$$= \oint du\left[\frac{f^{--}}{Q_\theta^-} + \frac{f^{++}}{Q_\theta^+} - \frac{\tau_1}{Q_\theta^+ Q_\theta^-}f\right]g$$

We can introduce L such brackets $\langle f\rangle_j = \oint du\, \mu_j\, f$

$$\langle f\hat{O}g\rangle_j = \langle g\hat{O}f\rangle_j \qquad \mu_j = e^{2\pi(j-1)u} \qquad j=1,\dots,L \qquad \tau_1 = 2\cos\phi\, u^L + \sum_{k=1}^{L} I_k u^{k-1}$$

uniquely identify the state

This gives orthogonality!

$$\langle Q^B(\hat{O}^A - \hat{O}^B)Q^A\rangle_j = 0 \qquad \hat{O} = \frac{1}{Q_\theta^+}D^2 + \frac{1}{Q_\theta^-}D^{-2} - \frac{\tau_1}{Q_\theta^+ Q_\theta^-}$$

$$\sum_{k=1}^{L}(I_k^A - I_k^B)\left\langle \frac{u^{k-1}Q^A Q^B}{Q_\theta^+ Q_\theta^-}\right\rangle_j = 0$$

Nontrivial solution means det=0

Sum of residues at $u=\theta_n \pm i/2$
i.e. at x eigenvalues as expected

$$\det_{1\le j,k\le L}\left\langle \frac{u^{k-1}Q^A Q^B}{Q_\theta^+ Q_\theta^-}\right\rangle_j \propto \delta_{AB}$$

Scalar product in SoV

Matches known results [Kitanine, Maillet, Niccoli, …]
[Kazama, Komatsu, Nishimura, Serban, Jiang, …]

SU(3) spin chain

For SU(3) we have 2 types of Bethe roots

$$\prod_{n=1}^{L}\frac{u_j-\theta_n-i/2}{u_j-\theta_n+i/2}=e^{i(\phi_1-\phi_2)}\prod_{k\neq j}^{N_u}\frac{u_j-u_k+i}{u_j-u_k-i}\prod_{l=1}^{N_v}\frac{u_j-v_l-i/2}{u_j-v_l+i/2}$$

$$1=e^{i(\phi_2-\phi_3)}\prod_{k\neq j}^{N_v}\frac{v_j-v_k+i}{v_j-v_k-i}\prod_{l=1}^{N_u}\frac{v_j-u_l-i/2}{v_j-u_l+i/2}$$

momentum-carrying $\{u_j\}_{j=1}^{N_u}$

auxiliary $\{v_j\}_{j=1}^{N_v}$

$$Q_1=e^{\phi_1 u}\prod_{j=1}^{N_u}(u-u_j) \qquad Q_{12}=e^{(\phi_1+\phi_2)u}\prod_{j=1}^{N_v}(u-v_j)$$

Main new feature: should use Q^i in addition to Q_i to get simple measure

Other Qs give dual roots $Q^1\equiv Q_{23},\ \mathrm{etc}$

Baxter equations:

$$\tau_a(u) = u^L \chi_a(G) + \sum_{j=1}^{L} u^{j-1}\, I_{a,j-1},$$

$$\bar{O} = \frac{1}{Q_\theta^-} D^{-3} - \frac{\tau_2}{Q_\theta^+ Q_\theta^-} D^{-1} + \frac{\tau_1}{Q_\theta^+ Q_\theta^-} D - \frac{1}{Q_\theta^+} D^{+3}$$

$$O = \frac{1}{Q_\theta^{++}} D^{+3} - \frac{\tau_2^+}{Q_\theta^{++} Q_\theta} D + \frac{\tau_1^-}{Q_\theta Q_\theta^{--}} D^{-1} - \frac{1}{Q_\theta^{--}} D^{-3}$$

$$\bar{O} \circ Q^a = 0 \qquad O \circ Q_a = 0$$

$$\langle f \rangle_j = \oint du\, \mu_j\, f$$

These two operators are conjugate! $\langle f O \circ g \rangle_j = \langle g \bar{O} \circ f \rangle_j$

$$\mu_j = e^{2\pi (j-1) u}$$

$$j = 1, \ldots, L$$

$$\langle Q_b^B (\bar{O}^A - \bar{O}^B) Q^{a,A} \rangle_j = 0$$

$$\tau_a(u) = u^L \chi_a(G) + \sum_{j=1}^{L} u^{j-1}\, I_{a,j-1},$$

$$\bar{O} = \frac{1}{Q_\theta^-} D^{-3} - \frac{\tau_2}{Q_\theta^+ Q_\theta^-} D^{-1} + \frac{\tau_1}{Q_\theta^+ Q_\theta^-} D - \frac{1}{Q_\theta^+} D^{+3}$$

$$\langle Q_b^B (\bar{O}^A - \bar{O}^B) Q^{a,A} \rangle_j = 0$$

We have freedom which Qs to choose

Linear system:

$$\sum_{\alpha=\{1,2\},\ k=1,\ldots,L} (I^A_{\alpha,k} - I^B_{\alpha,k})(-1)^\alpha \left\langle \frac{u^k Q_1^B Q^{a,A[-3+2\alpha]}}{Q_\theta^+ Q_\theta^-} \right\rangle_j = 0$$

We have 2L variables, and two choices of a give 2L equations

[Gromov, FLM, Ryan, Volin 19]

$$\langle \Psi_B | \Psi_A \rangle \propto \begin{vmatrix} \left\langle \frac{1}{Q_\theta^+ Q_\theta^-} u^{k-1} Q_1^A Q_B^{2+} \right\rangle_j & \left\langle \frac{1}{Q_\theta^+ Q_\theta^-} u^{k-1} Q_1^A Q_B^{2-} \right\rangle_j \\ \left\langle \frac{1}{Q_\theta^+ Q_\theta^-} u^{k-1} Q_1^A Q_B^{3+} \right\rangle_j & \left\langle \frac{1}{Q_\theta^+ Q_\theta^-} u^{k-1} Q_1^A Q_B^{3-} \right\rangle_j \end{vmatrix}$$

$$1 \leq j, k \leq L$$

Each bracket is a sum of residues at $u = \theta_n \pm i/2$

$$N_A^2 \delta_{AB} = \sum_{x,y} M_{x,y} \prod_{k=1}^{L} Q_1^A(X_{k,1}) Q_1^A(X_{k,2}) \prod_{k=1}^{L} \left[Q_B^2(Y_{k,1}) Q_B^3(Y_{k,2}) - Q_B^2(Y_{k,2}) Q_B^3(Y_{k,1}) \right]$$

matches spectrum of $B(u)$!

Can we build the basis where these are the wavefunctions?

Operator realization for SU(3)

[Gromov, FLM, Ryan, Volin 19]

$$\langle\Psi_B|\Psi_A\rangle = \int \left(\prod_{a=1}^{N-1}\prod_{i=1}^{L} dx_{i,a}\right) \left(\underbrace{\prod_{a=1}^{N-1}\prod_{i=1}^{L} Q_1^{(A)}(x_{i,a})}_{\text{state A}}\right) \hat{M}(\mathbf{x}) \left(\underbrace{\prod_{a=1}^{N-1}\prod_{i=1}^{L} Q^{(B)^a}(x_{i,a})}_{\text{state B}}\right)$$

$$\langle x|\Psi_A\rangle \qquad \langle \Psi_B|y\rangle$$

Instead of integrals we have sums

$$\langle\Psi_B|\Psi_A\rangle = \sum_{x,y} M_{x,y}\langle\Psi_B|y\rangle\langle x|\Psi_A\rangle$$

Get scalar product from construction of two SoV bases $|y\rangle$ and $\langle x|$

[Sklyanin 92] [Gromov FLM Sizov 16]

$\langle x|$ are eigenstates of familiar operator $\hat{\mathbb{B}}(u) = \hat{T}^2{}_3(u)\hat{U}_3{}^1(u-i) - \hat{T}^1{}_3(u)\hat{U}_3{}^2(u-i)$

$|y\rangle$ are eigenstates of new "dual" operator $\hat{\mathbb{C}}(u) = \hat{T}^2{}_3(u-\tfrac{i}{2})\hat{U}_3{}^1(u-\tfrac{i}{2}) - \hat{T}^1{}_3(u-\tfrac{i}{2})\hat{U}_3{}^2(u-\tfrac{i}{2})$

$$M_{x,y} = (\langle x|y\rangle)^{-1}$$

Measure matches what we got from Baxter!

To build SoV basis we act on reference state with transfer matrices

B(u) is diagonalized by [Maillet, Niccoli 18] [Ryan, Volin 18]

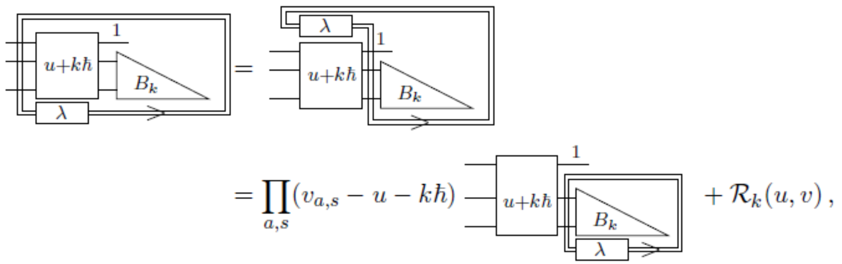
$$\langle x| \propto \langle 0| \prod_{k=1}^{L} [\hat{\tau}_2(\theta_k - i/2)]^{m_{k,1}+m_{k,2}} \qquad 0 \le m_{k,1} \le m_{k,2} \le 1$$

C(u) is diagonalized by [Ryan, Volin 18] [Gromov FLM, Ryan, Volin 19]

$$|y\rangle \propto \prod_{k=1}^{L} \hat{\tau}_1(\theta_k - i/2)^{n_{k,2}-n_{k,1}} \; \hat{\tau}_2(\theta_k - i/2)^{n_{k,1}} |0\rangle \qquad 0 \le n_{k,1} \le n_{k,2} \le 1$$

Proof is direct generalization of
highly nontrivial methods from [Ryan, Volin 18]

Based on commutation relations +
identifying Gelfand-Tsetlin patterns



$$M_{x,y} = (\langle x|y\rangle)^{-1}$$

$$\langle \Psi_B|\Psi_A\rangle = \sum_{x,y} M_{x,y} \langle \Psi_B|y\rangle\langle x|\Psi_A\rangle$$

Notice for SU(2) the overlaps matrix is diagonal

For SU(3) it is not, but the elements are still simple!

$$\langle \Psi_B|\Psi_A\rangle \propto \begin{vmatrix} \left\langle \frac{1}{Q_\theta^+ Q_\theta^-} u^{k-1} Q_1^A Q_B^{2+} \right\rangle_j & \left\langle \frac{1}{Q_\theta^+ Q_\theta^-} u^{k-1} Q_1^A Q_B^{2-} \right\rangle_j \\ \left\langle \frac{1}{Q_\theta^+ Q_\theta^-} u^{k-1} Q_1^A Q_B^{3+} \right\rangle_j & \left\langle \frac{1}{Q_\theta^+ Q_\theta^-} u^{k-1} Q_1^A Q_B^{3-} \right\rangle_j \end{vmatrix}$$

[Gromov, FLM, Ryan, Volin 19]

Alternative approach: [Maillet, Niccoli, Vignoli 20]
fix measure indirectly by deriving recursion relations for it

Diagonal form factors of type $\frac{\langle\Psi|\frac{\partial \hat{I}_n}{\partial p}|\Psi\rangle}{\langle\Psi|\Psi\rangle} = \frac{\partial I_n}{\partial p}$ are computable, give ratios of determinants.

From self-adjoint property:

$$0 = \langle Q(\hat{O} + \delta O) \circ (Q + \delta Q)\rangle = \underbrace{\langle QO \circ \delta Q\rangle}_{= 0} + \langle Q\delta O \circ Q\rangle$$

$$\tau_1 = 2\cos\phi\, u^L + \sum_{k=0}^{L-1} I_k u^k$$

Link δI_n with $\delta\phi$

So $$\partial_\phi I_k = \frac{1}{2\sin\phi} \frac{\det_{i,j=1,\dots,L} m_{ij}^{(k)}}{\det_{i,j=1,\dots,L} m_{ij}}$$

norm

All this generalizes to SU(N)

Algebraic picture

Generating functional for transfer matrices in antisymmetric reps

$$W = (1-\Lambda_1(u)D^2)(1-\Lambda_2(u)D^2)\dots(1-\Lambda_N(u)D^2) = \sum_{k=1}^{N}(-1)^k\tau_k(u)D^k$$

Define left and right action $\overrightarrow{D}f(u) = f(u+i/2), \quad f\overleftarrow{D} = f(u-i/2)$

Then $Q_a\overleftarrow{W} = 0$ and $\overrightarrow{W}Q^a = 0$

Using that for any operator $\oint g\overrightarrow{O}f = \oint f\overleftarrow{O}g$ we get $\oint Q_a^A(\overrightarrow{W}_A - \overrightarrow{W}_B)Q_B^b = 0$

Comment on chronology:

Such tricks with Baxters were used in [Cavaglia, Gromov, FLM 18] for cusp

Then in [Cavaglia, Gromov, FLM 19] for SL(N) spin chain

And then in [Gromov, FLM, Ryan, Volin 19] for SU(N) spin chain

NON-COMPACT SPIN CHAINS

[Cavaglia, Gromov, FLM 19]

Infinite-dim highest weight representation of SL(N) on each site

Now we have integrals instead of sums $\langle f\rangle_j = \int_{-\infty}^{\infty} du\, \mu_j\, f$ $\quad \mu_j = \frac{1}{1+e^{2\pi(u-\theta_j)}}$

$\bar{O}\circ Q^a = 0$ $\quad O\circ Q_a = 0$

$$\bar{O} = Q_\theta^- D^{-3} - \tau_2 D^{-1} + \tau_1 D - Q_\theta^+ D^{+3}$$

$$O = Q_\theta^{++} D^{+3} - \tau_2^+ D + \tau_1{}^- D - Q_\theta^{--} D^{-3}$$

We would like $\langle g\bar{O}\circ f\rangle = \langle fO\circ g\rangle$

Now when we shift the contour we cross poles of the measure

$$\langle g\bar{O}\circ f\rangle = \int \mu\, g\left[Q_\theta^- f^{[-3]} - \tau_2 f^- + \tau_1 f^+ - Q_\theta^+ f^{[+3]}\right] = \langle fO\circ g\rangle + \text{pole contributions}$$

$$Q_1(\theta_j+\tfrac{i}{2})\tau_1(\theta_j+\tfrac{i}{2}) - Q_1(\theta_j+\tfrac{3i}{2})Q_\theta(\theta_j+\tfrac{i}{2}) = 0$$

Poles cancel when $g = Q_1$! Then everything works as before

[Cavaglia, Gromov, FLM 19]

General structure in SL(N):

$$\langle\Psi_A|\Psi_B\rangle = \int \left(\prod_{a=1}^{N-1}\prod_{i=1}^{L} dx_{i,a}\right)\left(\underbrace{\prod_{a=1}^{N-1}\prod_{i=1}^{L} Q_1^{(A)}(x_{i,a})}_{\text{state A}}\right)\hat{M}(\mathbf{x})\left(\underbrace{\prod_{a=1}^{N-1}\prod_{i=1}^{L} Q^{(B)^a}(x_{i,a})}_{\text{state B}}\right)$$

state-independent operator, contains shifts

$$\hat{M}(x) = \det\left|\underbrace{\left(\frac{\hat{x}^{j-1}}{1+e^{2\pi(\hat{x}-\theta_i)}}\right)}_{1\leqslant i,j\leqslant L}\otimes\underbrace{\begin{pmatrix}\mathcal{D}_x^{N-2} & \mathcal{D}_x^{N-4} & \dots & \mathcal{D}_x^{2-N}\\ \vdots & \vdots & \ddots & \vdots\\ \mathcal{D}_x^{N-2} & \mathcal{D}_x^{N-4} & \dots & \mathcal{D}_x^{2-N}\end{pmatrix}}_{(N-1)\times(N-1)}\right|$$

similar to conjecture of [Smirnov Zeitlin]
based on semi-classics
and quantization of alg curve

We also generalized to any spin s of the representation [Gromov FLM, Ryan, Volin to appear]

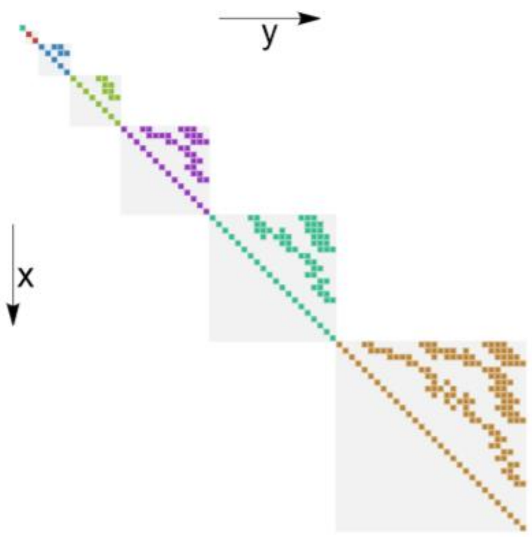
$$\langle f\rangle_n = \int_{-\infty}^{\infty} du\,\mu_n\,f \qquad \mu_n = \frac{1}{1+e^{2\pi(u-\theta_n)}} \Rightarrow \mu_n = \frac{\Gamma(s-i(u-\theta_n))\Gamma(s+i(u-\theta_n))}{e^{\pi(u-\theta_n)}}$$

For SL(2) we reproduce [Derkachov, Manashov, Korchemsky]

To build SoV basis we need more involved T's in non-rectangular reps see [Ryan, Volin 20]

$$|x\rangle \propto \hat{T}_{\{m_1,m_2\}}\left(\theta_n + is + i\frac{m_1-\mu_1'}{2}\right)|0\rangle$$

Integral = sum over infinite set of poles in lower half-plane



The measure we get from Baxters again matches the one from building the basis!

FISHNET THEORY

Gamma-deformed N=4 SYM:

$$\mathcal{L}_{\rm int} = N_c\, g^2\, {\rm tr}\left(\frac{1}{4}\{\phi_i^\dagger,\phi^i\}\{\phi_j^\dagger,\phi^j\} - e^{-i\epsilon^{ijk}\gamma_k}\phi_i^\dagger\phi_j^\dagger\phi^i\phi^j\right)$$

Frolov Roiban Tseytlin 05

$\gamma_1,\ \gamma_2,\ \gamma_3$ are 3 deformation parameters, no susy but integrable

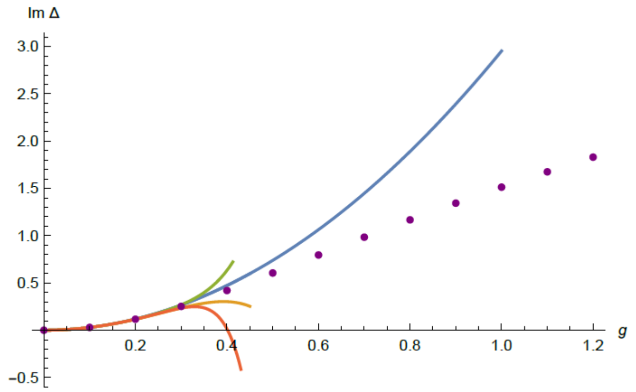
Still have integrability, e.g. for operator ${\rm tr}(\phi_1\phi_1)$ QSC gives

FLM, Preti 20

$$\begin{aligned}\Delta &= 2\\ &+ 8i\sin\frac{\gamma_1}{2}\sin\frac{\gamma_2}{2}g^2\\ &+ 0\times g^4\\ &+ \left[48i\zeta(3)\sin\frac{\gamma_1}{2}\sin\frac{\gamma_2}{2}(\cos\gamma_1+\cos\gamma_2-2) - 64i\sin^3\frac{\gamma_1}{2}\sin^3\frac{\gamma_2}{2}\right]g^6\\ &+ \left[1024i\zeta(3)\sin^3\frac{\gamma_1}{2}\sin^3\frac{\gamma_2}{2} - 640i\zeta(5)\sin\frac{\gamma_1}{2}\sin\frac{\gamma_2}{2}(\cos\gamma_1+\cos\gamma_2-2)\right]g^8\\ &+ \ldots\end{aligned}$$

feels mixing with double traces

Fokken, Sieg, Wilhelm 14



Gamma-deformed N=4 SYM:

$$\mathcal{L}_{\text{int}} = N_c\, g^2 \,\text{tr}\left(\frac{1}{4}\{\phi_i^\dagger,\phi^i\}\{\phi_j^\dagger,\phi^j\} - e^{-i\epsilon^{ijk}\gamma_k}\phi_i^\dagger\phi_j^\dagger\phi^i\phi^j\right)$$

Frolov Roiban Tseytlin 05

$\gamma_1,\ \gamma_2,\ \gamma_3$ are 3 deformation parameters, no susy but integrable

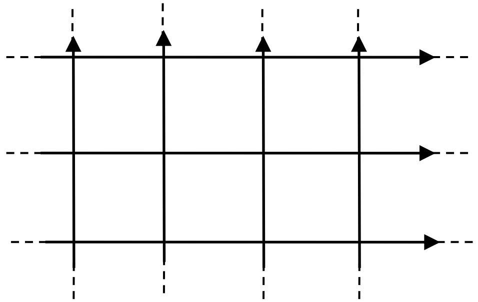
Consider the limit: strong twist, weak coupling $g \to 0, \qquad e^{-i\gamma_j/2} \to \infty, \qquad \xi_j = g\, e^{-i\gamma_j/2}$ – fixed, $\qquad (j = 1,2,3.)$

Result known as fishnet theory

Gurdogan, Kazakov 15

$$\mathcal{L}[\phi_1,\phi_2] = \frac{N}{2}\text{tr}\left(\partial^\mu\phi_1^\dagger\partial_\mu\phi_1 + \partial^\mu\phi_2^\dagger\partial_\mu\phi_2 + 2\xi^2\,\phi_1^\dagger\phi_2^\dagger\phi_1\phi_2\right).$$

Feynman diagrams are "fishnet" graphs



Zamolodchikov 81

Inherits integrability and the QSC

Gromov, Kazakov,
Korchemsky, Negro, Sizov 17

Dual model = 'discretized string' fishchain

Gromov, Sever 19
see also Basso, Zhong

Gromov, Kazakov, Korchemsky; Caetano, Gurdogan, Kazakov; Ibsen, Staudacher, Zippelius;
Basso, Dixon; Derkachov, Olivucci, Preti, …

We twist the AdS space-time symmetries to remove degeneracies

Cavaglia, Grabner, Gromov, Sever 20

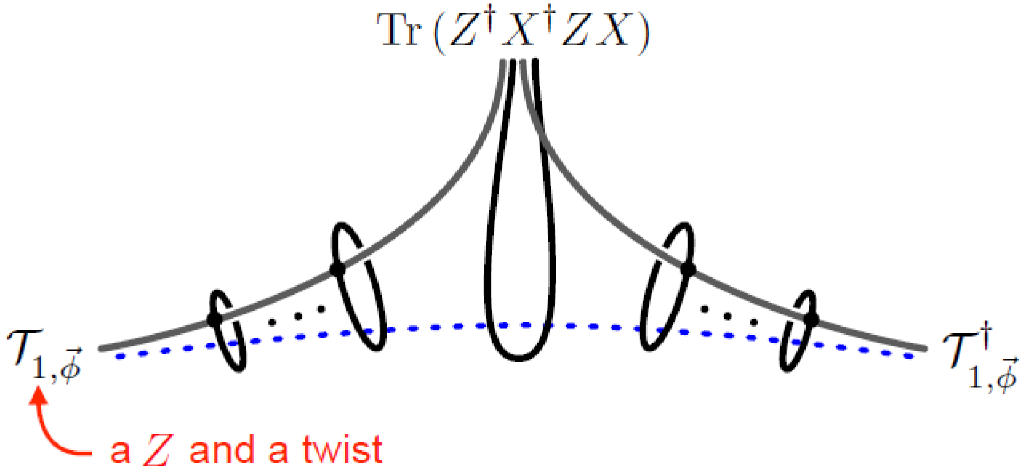
Becomes quite similar to cusp

We study operators Tr (ϕ_1^L)

For L=1 use same tricks with QSC Baxter equation

For the simplest 3 pt function: $\langle \mathcal{O}\mathcal{O}\mathcal{L}\rangle$

$$\partial_{\xi^2}\Delta = \frac{\langle\Psi|\mathcal{L}|\Psi\rangle}{\langle\Psi|\Psi\rangle} = \frac{\int_{|} \frac{du}{u} q_1(u)\bar{q}_{\bar{1}}(u)}{2i\sin\frac{\phi_1}{2}\sin\frac{\phi_2}{2}\int_{|} du\, q_1(u)(\bar{q}_{\bar{1}}^{++}(u) - \bar{q}_{\bar{1}}^{--}(u))}$$



We also found explicit map: diagrams ⟺ Q-functions

Cavaglia, Grabner, Gromov, FLM, Sever to appear

Extension to higher L in progress, Baxter is known

Gromov, Sever 19

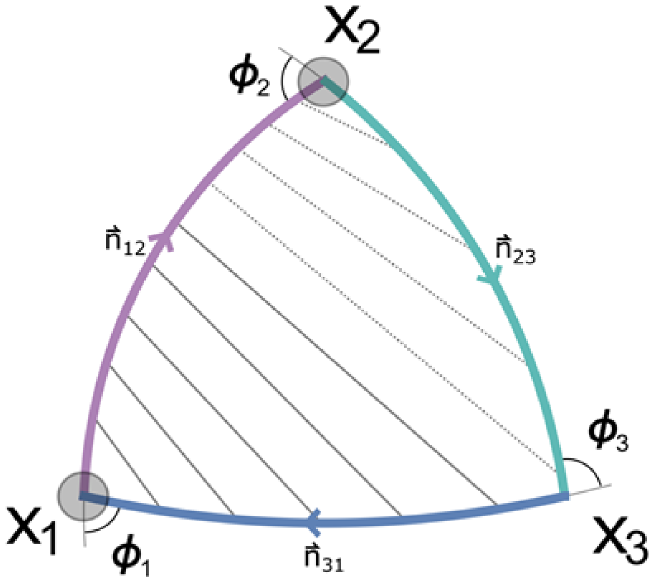
3-cusp correlator done so far with L=0 insertions
(same scalars as coupled to the line) or slight generalization

[Cavaglia, Gromov, FLM 18]

[McGovern 20]

$$C_{123}^{\bullet\bullet\circ} = \frac{\langle q_1\, q_2\, e^{-\phi_3 u}\rangle}{\sqrt{\langle q_1^2\rangle\langle q_2^2\rangle}}$$

$$W = \mathrm{Tr}\,\mathcal{P}\exp\int dt\left[iA\cdot\dot{x} + \vec{\Phi}\cdot\vec{n}\,|\dot{x}|\right]$$



Baxter equation was recently understood for higher L
Should help to do more general insertions

[Gromov, Julius 20
+ to appear]

The goal is to gather data from all these examples
to attack the full N=4 SYM

FUTURE

- Finally we know SoV measure for higher-rank spin chains; encouraging results for fishnets
- Extensions: super case [Gromov, FLM 18], SO(N) [Ferrando, Frassek, Kazakov; Ekhamar, Shu, Volin 20], principal series rep for fishnet, Slavnov scalar products
- More general correlators for fishnet & cusps, corrections to fishnet
- Links with hexagons & SoV of [Derkachov, Olivucci, Basso, Kazakov, Ferrando, Zhong]
- QSC for g-function TBA? [Jiang, Komatsu, Vescovi 19]
  Applications for SU(N) PCF? [talk of Evgeny Sobko]